CSCI 246: Test 3 (10 points, 4:10-5:30pm)

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Note: If you don't have a printer, you should try to write/type separately. At the end of the test, try to generate a .pdf file to upload to D2L (under Assignments/Test 2). *Note also that this is an open book test, while all physical resources are allowed, resorting for external human help constitutes a* **plagiarism.**

Problem 1

In this problem, you are given *four* mathematical statements. You are first to judge their correctness, in each case you should give some details (like a short proof or a counterexample) to support your answers.

(1.1) $2020 \cdot n^2$ is $\Omega(n^3)$. False

for $n > 1$ and $A = 2020$, $2020 \cdot n^2 < An^3$

$\therefore 2020n^2$ is not at least An^3

(1.2) $2020 \cdot n^3$ is $O(n^2)$. False

for $n > 1$ and $A = 2020$, $2020 \cdot n^3 > An^2$

$\therefore 2020n^3$ is not at most An^2

(1.3) Let G be a simple graph (i.e., between two vertices there is at most one edge), and let $deg(v)$ be the degree of vertex v. Then $\sum_{v \in V(G)} deg(v)$ is always even.

True

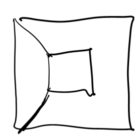
Because G is simple, an edge cannot connect a vertex to itself. A graph with 0 edges has a sum of degrees of zero. For every edge added, the degree of each connecting vertex goes up by 1.

$\therefore$ Any edge corresponds to an increase of 2 in the sum of vertex degrees.

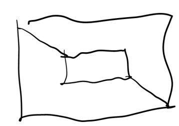
(1.4) Given simple graphs G and H, both with n vertices. Sort the vertex degrees (from small to large) in G and H as L_G and L_H respectively. If $L_G = L_H$, then G and H must be isomorphic.

False

$G =$



$H =$



$L_G = 3, 3, 3, 3, 2, 2, 2, 2$

$L_H = 3, 3, 3, 3, 2, 2, 2, 2$

Problem 2

Show that $f(n) = \frac{1}{6}n^3 - 32n^2 + 20n - 8$ is $\Theta(n^3)$. You must give enough details.

by Theorem of polynomials with positive exponents,

$f(n)$ is $\Theta(n^3)$

$A = \frac{1}{12}$ and $a = \frac{2 \cdot 60}{\frac{1}{6}} = 240$

$f(n) \geq \frac{1}{6}n^3 - 32n^2 + 20n^2 - 8n^2 = \frac{1}{6}n^3 - 20n^2 \geq \frac{1}{12}n^3$

when $n \geq 720$

$\therefore$ $f(n)$ is $\Omega(n^3)$

$f(n) \leq 1/6\, n^3 + 20n \leq 1/6\, n^3 + 20n^3$ when $n \geq 1$

$1/6\, n^3 + 20n^3 = \frac{121}{6} n^3$

set $A = \frac{121}{6}$ and $a = 1$

$f(n) \leq An^3$ when $n \geq 9$

$\therefore$ $f(n)$ is $O(n^3)$

$\therefore$ $f(n)$ is $\Theta(n^3)$

Problem 3

Let S be the set of integers from 20,000 to 99,999 (including 20,000 and 99,999).

(3.1) How many integers are there in S which are divisible by 3 and 7, but not by both?

$$\left\lfloor \frac{99{,}999}{3} \right\rfloor - \left\lfloor \frac{19{,}999}{3} \right\rfloor = 26{,}667$$

$$\left\lfloor \frac{99{,}999}{7} \right\rfloor - \left\lfloor \frac{19{,}999}{7} \right\rfloor = 11428$$

$$\left\lfloor \frac{99{,}999}{21} \right\rfloor - \left\lfloor \frac{19{,}999}{21} \right\rfloor = 3809$$

$$26{,}667 + 11428 - 2(3809) = 30477$$

(3.2) How many even integers are there in S which are with distinct digits?

$99{,}999 - 20{,}000 + 1 = 80{,}000$

even in S = $80{,}000/2 = 40{,}000$

$5 \cdot 7 \cdot 8 \cdot 7 \cdot 6 = 11760$

last first

12096? got with a python script

Problem 4

Two different factories both produce a certain automobile part. The probability that a component from the first factory is defective is 4%, and the probability that a component from the second factory is defective is 6%. In a supply of 200 of the parts, 150 were obtained from the first factory and 50 from the second factory.

(4.1) What is the probability that a part chosen at random from the 200 is defective?

75% chance came from first.

25% chance came from second

$$\underbrace{0.75 \cdot 0.04}_{\text{first factory defective}} + \underbrace{0.25 \cdot 0.06}_{\text{second factory defective}} = 4.5\%$$

(4.2) If the chosen part is defective, what is the probability that it came from the first factory?

$$P(\text{first fact} \mid \text{defective}) = \frac{P(\text{defect} \mid \text{first})P(\text{first})}{P(\text{defect} \mid \text{first}) \cdot P(\text{first}) + P(\text{defect} \mid \text{second}) P(\text{second})}$$

$$= \frac{0.04 \cdot 0.75}{0.04 \cdot 0.75 + 0.06 \cdot 0.25} = 66.7\%$$