CSCI 246: Test 2 (10 points, 4:10-5:20pm)

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Note: If you don't have a printer, you should try to write/type separately. At the end of the test, try to generate a .pdf file to upload to D2L (under Assignments/Test 2). *Note also that this is an open book test, while all physical resources are allowed, resorting for external human help constitutes a* **plagiarism.**

Problem 1

(1.1) Calculate the summation

$$\sum_{i=5}^{n} \frac{1}{i \cdot (i+1)} = \frac{1}{5 \cdot 6} + \frac{1}{6 \cdot 7} + \cdots + \frac{1}{n \cdot (n+1)}.$$

$$\sum_{i=1}^{n} \frac{1}{i \cdot (i+1)} = \frac{n}{n+1}$$

$$\sum_{i=5}^{n} \frac{1}{i \cdot (i+1)} = \frac{n}{n+1} - \sum_{i=1}^{4} \frac{1}{i \cdot (i+1)} = \frac{n}{n+1} - \left(\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20}\right)$$

$$= \frac{n}{n+1} - \frac{4}{5}$$

(1.2) How many zeros are at the end of $45^{10} \cdot 88^5$? Explain how you can answer this question without actually computing the number.

$45 = 5 \cdot 9 = 5 \cdot 3 \cdot 3$

$88 = 2 \cdot 44 = 2 \cdot 2 \cdot 22 = 2 \cdot 2 \cdot 2 \cdot 11$

$(45)^{10} = (5 \cdot 3 \cdot 3)^{10} = 5^{10} 3^{20}$

$(88)^5 = (2^3\, 11)^5 = (2^{15}\, 11^5)$

ans = 10

$45^{10} \cdot 88^5 = 5^{10}\, 3^{20}\, 2^{15}\, 11^5$

factors of 10 are 5 and 2, so for every 5,2 pair in the prime factorization of the number, we add one 0.

We have 10 (5,2) pairs, so 10 zeros.

Problem 2

Prove that $\sqrt{7}$ is irrational.

assume that $\sqrt{7}$ is rational

let a, b be integers with $b \neq 0$ and a, b are co-primes so no common factors $\neq 1$

$\sqrt{7} = \frac{a}{b}$ $\qquad 7 = \frac{a^2}{b^2}$

$7b^2 = a^2$ / therefore a is divisible by 7

let $a = 7k$

$7b^2 = (7k)^2$

$7b^2 = 49k^2$

$b^2 = 7k^2$ / therefore b is also divisible by 7

Contradiction with the premise that a and b have greatest common denominator of 1

$\therefore$ assuption is false, so $\sqrt{7}$ must be irrational

Problem 3

Prove that $8^n - 3^n$ is divisible by 5 for every integer $n \geq 0$.

let x be an arbitrary positive integer

let $8^n - 3^n = 5(x)$

basis: $8^0 - 3^0 = 0$ $\quad 8^1 - 3^1 = 5$ $\quad 8^2 - 3^2 = 55$

IH: Assume that $8^n - 3^n = 5x$ is true for $0 \leq i \leq k$

IS: $8^{n+1} - 3^{n+1} = 5x$

$= 8 \cdot 8^n - 3 \cdot 3^n = 5x$

$= (5+3) \cdot 8^n - 3 \cdot 3^n = 5 \cdot 8^n + 3 \cdot 8^n - 3 \cdot 3^n$

$= 5 \cdot 8^n + 3(8^n - 3^n)$

$= 5 \cdot 8^n + 3(5(x))$ / by IH

$= 5(8^n + 3x)$

$\therefore$ divisible by 5

Problem 4

Solve the following recurrence relation and prove the correctness of your solution by induction.

$T(n) = 2T(n/4) + 2n^2,$
$T(1) = 1.$

$$T(n) = 2T(n/4) + 2n^2$$
$$= 2\left[2T\left(\frac{n}{4^2}\right) + 2\left(\frac{n}{4}\right)^2\right] + 2n^2$$
$$= 4\left(T\left(\frac{n}{4^2}\right)\right) + \frac{n^2}{4^2} + 2n^2$$
$$= 4\left[2T\left(\frac{n}{4^3}\right) + 2\left(\frac{n}{4^2}\right)^2\right] + \frac{n^2}{4^2} + 2n^2$$
$$= 8\left[T\left(\frac{n}{4^3}\right)\right] + \frac{n^2}{4^3} + \frac{n^2}{4^2} + 2n^2$$

guess $= 2^i T\left(\frac{n}{4^i}\right) +$

$\frac{n}{2^i} = 1$